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This research has focussed on the development of feasible approaches to the utilization of computer algebra to derive the Green's Function of interconnected structural systems. The key objectives of the effort were to: (1) develop MACSYMA-based routines for analyzing the Green's functions of structural systems consisting of uniform beams supported at discrete locations by linear springs; and (2) extend these techniques to include attachments of linear spring-mass-damper substructures to uniform beams. Both of these objectives were met. This final report summarizes the methodologies that were developed, including appendices which present the MACSYMA routines illustrating the application of these methodologies, as well as a paper which was presented in Singapore at the International Noise and Vibration '89 Conference.

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SUMMARY

This final technical report summarizes the research carried out under sponsorship of AFOSR contract number F49620-88-C-0060, awarded to AEDAR Corporation for the period May 1, 1988 - July 31, 1989. The main thrust of the research has been the establishment of feasible approaches for using computer algebra to derive the Green's Function of interconnected structural systems. The accomplishments of the research effort during this period include the formulation, analysis and coding of computer-algebraic techniques for the derivation of the Green's functions of structural systems consisting of uniform Euler-Bernoulli beams which are either supported elastically or to which discrete substructures have been attached. The computer-algebraic environment used was MACSYMA. Using a general representation of the Green's function of elastic structures, the process of deriving the Green's function of such interconnected structures consisted of the following stages: (1) Solution of the integral equation which determines the transfer function of the updated system, (2) Determination of the characteristic parameters of the transfer function for the updated system, (3) Determination of the characteristic functions of the transfer function for the updated system, (4) Representation of the transfer function for the updated system as a series, and (5) Laplace inversion of the transfer function.

FOREWORD

This report was prepared by the AEDAR Corporation, under the Air Force Office of Scientific Research contract number F49620-88-C-0060. The work was performed by Dr. James A. Fabunmi during the period May 1, 1988 and July 31, 1989.

The author wishes to thank Dr. Anthony K. Amos for his support of this research.

TABLE OF CONTENTS

I. OBJECTIVES OF THE RESEARCH

The objectives of the research effort were:

(1) To develop MACSYMA-based routines for analyzing the Green's functions of structural systems consisting of uniform beams supported at discrete locations by linear springs.

(2) To extend these techniques to include attachments of linear spring-mass-damper substructures to uniform beams.

II. STATUS OF THE RESEARCH EFFORT

After exploring different methods available for the computer-algebraic representation of the Green's function for flexible structures, a formulation was obtained which allowed the automatic update of the characteristic parameters and characteristic functions by means of which the new Green's functions of the modified structure can be represented. First the derivations are accomplished on the basis of the Laplace transform of the Green's function, otherwise known as the transfer function, then the inverse Laplace transform of the transfer function is taken in order to yield the Green's function. A truncated modal series is used to represent the transfer function.

The characteristic parameters of the updated transfer function were obtained as the generally complex roots of a characteristic polynomial. Using the baseline parameters as a starting approximation, an iterative Newton-Raphson scheme was used to search for the roots of the polynomial. The issues of convergence of the truncated series and that of the precision of the identified roots were investigated. It was found that the computer routine for performing these derivations can be structured in such a way that any desired precision can be achieved, at the expense of retaining many terms in the truncated series and using multiple stages in the iterative search. Because of limitations in the existing computational facility for performing these research tasks, further investigation of these issues have been deferred to the following year of the effort, since a dedicated Symbolics Workstation has been requested for this purpose.

Using a general representation of the Green's function of elastic structures, the process of deriving the Green's function of such interconnected structures consisted of the following stages: (1) Solution of the integral equation which determines the transfer function of the updated system, (2) Determination of the characteristic parameters of the transfer function for the updated system, (3) Determination of the characteristic functions of the transfer function for the updated system, (4) Representation of the transfer function for the updated system as a series, and (5) Laplace inversion of the transfer function.

III. LIST OF PUBLICATIONS

Fabunmi, J.A., "Analysis of Modes and Frequencies of Modified Structures Using Computer Algebra", Proceedings of the International Conference on Noise and Vibration '89, Nayang Technological Institute, Singapore, August 16-18, 1989. (See Appendix IV).

IV. PROFESSIONAL PERSONNEL ASSOCIATED WITH RESEARCH EFFORT

Dr. James A. Fabunmi.

V. INTERACTIONS

(1) Clark Atlanta University: Joint proposal effort with Kwabena B. Bota - AFOSR Proposal #89-NA-042 entitled "Analysis of Active Controller Effects on Flexible Structures Using Computer Algebra".

VI. ADDITIONAL COMMENTS

The formulations which have been obtained for the interconnection of uniform beams to discrete substructures have turned out to be more general than was originally anticipated. Any one-dimensional structure for which the characteristic functions and characteristic parameters are available, can replace the uniform beam. Thus nonuniform beams for which a separate analysis and/or experimental tests have been used to obtain this information, can be treated. The discrete substructure is likewise not limited to spring-mass-dampers only. Any attachment for which an explicit transfer function is available in the form of a ratio of polynomials in the Laplace variable, can also be handled.

At the moment, the theoretical concepts for dealing with discrete attachments to one-dimensional baseline structures appear to be fully developed. The computer implementation of these ideas is somewhat lagging due to the difficulties that have been encountered in using the MACSYMA on the VAX via a remote terminal. It is expected that the dedicated Symbolics Workstation which has been requested in the follow-on proposal, will alleviate these difficulties considerably. Appendices I - III are the listings of the MACSYMA codes which have been developed so far. Appendix IV is a copy of the paper that was presented at the International Noise & Vibration '89 Conference in Singapore. This paper presents details of the formulation of the methodologies which have been developed during this research effort.

APPENDIX I - Listing of MACSYMA routine GF1D.MAC

```
comment():=(print("this module derives the characteristic
        parameter and characteristic function for a specified
        term in the Green's function series for the system
        comprising the baseline structure (batchloaded with the
        run) to which the modification (batchloaded with the
        run) has been attached at the location specified in the
        system setup routine. this program assumes that the
        baseline structure is one-dimensional and the
        modification is lumped parameter. to run the program,
        type -
                        domode(index)
        where index is the integer that defines the desired
        term in the series. "))$

    domode(index):=(numer:true,ratprint:false,
        system(),idxrange(index),iter(index),print("done"))$

    system():=block(print("commence system setup routine"),
        again1, bs:read("has baseline structure been defined?
        (yes/no)"),
        if string(bs)="no" then (load_baseline(),go(cont1))
        else(
        if string(bs)="yes" then go(cont1) else(
        print("inappropriate response"),go(again1))),cont1,
        bs:read("has sub-structure attachment been defined?
        (yes/no)"),
        if string(bs)="no" then (load_substr(),go(cont2)) else(
        if string(bs)="yes" then go(cont2) else(
        print("inappropriate response"),go(cont1))),cont2,
        print("system setup complete, commence analysis"))$

    load_baseline():=block(print("routine for loading baseline
        structure"),
        bsfile:read("enter filename for baseline structure
        (extension .mac is assumed)"),load(bsfile),
        base_structure())$

    load_substr():=block(print("routine for loading
        sub-structure
        attachment"),
        ssfile:read("enter filename for sub-structure
        attachment
        (extension .mac is assumed)"),load(ssfile),
        sub_structure(),
        bl:read("enter attachment point, as fraction of length
        of baseline structure"),b:bl*el)$


    idxrange(argk):=block(
        print("determine range of indices needed"),
            startp(argk),
```

```
          p[0]:p0[argk],val[0]:p[0],
          pz:realpart(p[0])+%i*imagpart(p[0]),
     print("char. param. for baseline str. for this mode =
          ", pz),
          icount:0,loop,icount:icount+1,
          itl:argk-icount,itr:argk+icount,
          if itl<1 then itl:1,
          for kk:itl thru itr do(startp(kk)),
          ntr:1,newton(1), val[icount]:p[1],
          iterms:itr-itl+1,
     print("for left index of ",itl," right index of ",itr,
          "i.e. no. of terms of ",iterms),
          pz:realpart(p[1])+%i*imagpart(p[1]),
     print("single iteration Newton gives mod. char. param.
          as", pz),

err:abs(val[icount]-val[icount-1])/abs(val[icount-1]),
     print("dimensionless difference = ",err),
          if err>0.001 then go(loop) )$

          iter(argk):=block(print("determine no. of Newton
          iterations"),

ntr:0,valpol[0]:poly(p[0]),bs1:abs(valpol[0])+.001,
          base:min(bs1,1),
          loop,ntr:ntr+1,
          for n:1 thru ntr do(newton(n)),
          valpol[ntr]:poly(p[ntr]),
     print("valpol = ",valpol[ntr]),
          err:abs(valpol[ntr])/base,
          if err>0.001 then go(loop) else
          p1[index]:p[ntr])$

newmode():=block(pnew:p1[index],for ichk:itl thru itr
          do(if pnew=p0[ichk] then
          (oldmode(ichk),go(exit))),
          wc1:ev(wc,p:pnew),
          wbp1:sum(phi(b,ks)**2/(pnew**2-p0[ks]**2),
          ks,itl,itr),if ev(wbp1)=0 then
          (brigid(),go(exit)),
          dnomial:expand(denwc*sum1),
          hk:ev(rempoly/dnomial,p:pnew),
          if hk=0 then (mtplrt(),go(exit)),
          for iold:itl thru itr do(
          top:2*pnew*phi(b,iold)**2*wc1,
          btm:(pnew**2-p0[iold]**2)**2*wbp1*hk,
          alpha2[index,iold]:top/btm),
          alpha[index,iold]:sqrt(alpha2[index,iold]),
          output(),exit)$

output():=(print("characteristic parameter
          p(",index,")=",pnew),
     print("index of lowest term used in series   =",itl),
     print("index of highest term used in series  =",itr),
```

```
      print("number of Newton iterations used =",ntr),
      for imode:itl thru itr do(
      print("contribution of mode ",imode,"to new mode =",
      alpha[index,imode]))$

mtplrt():=(print("characteristic polynomial has multiple
          roots for mode ",index," this case is not
          yet considered- end of analysis."))$

brigid():=(print("attachment point for substructure is
          rigid,
          dynamics of baseline structure is unchanged-
          end of analysis."))$

oldmode(oldindx):=(print("parameters of mode ",index," of
          modified structure coincide with those of
          mode ",oldindx," of baseline structure-
          end of analysis."))$

poly(pval):=(pr1:product(p**2-p0[kp]**2,kp,itl,itr),
          sum1:sum(phi(b,ks)**2*pr1/(p**2-p0[ks]**2),
               ks,itl,itr),
          pnomial:expand(denwc*pr1-numwc*sum1),
          nhi:hipow(pnomial,p),
          for nt:0 thru nhi do(a[nt]:ratcoef(pnomial,p,nt)),
          b[nhi]:a[nhi],
          for nt:nhi-1 step -1 thru 0
          do(b[nt]:a[nt]+pval*b[nt+1]),
          rempoly:sum(b[nt]*p**(nt-1),nt,1,nhi),
          b[0])$


newton(n):=block(print("doing the ",n,"th update of a ",ntr,
     "-round Newton iteration"),
     switch:0,nn:0,pm1:p[n-1],loop,
     ffprime(n),
     pp:pm1-f[0]/f[1],errp:abs(pp-pm1)/abs(pm1),
     switch:1,pm1:pp,nn:nn+1,
     if errp>.001 then go(loop),
     print(nn,"internal newton loops"),
     p[n]:pp)$

ffprime(n):=(if n=1 then (if switch<1 then route1() else
     route2())
     else route2())$

route1():=(
     ell:argk,phi(b,ell),
     exp0:-(1/ntr)*wc*phi**2,
     f[0]:ev(exp0,p:p[0]),
     sumj(ell,p[0]),
     exp1:2*p[0]-(1/ntr)*dwc*phi**2+exp0*sumj,
     f[1]:ev(exp1,p:p[0]))$
```

```
sumj(argl,argp):=(sumj:0,for j:itl thru itr do(
     if j=argl then term:0 else
     term:2*argp/(argp**2-p0[j]**2),
     sumj:sumj+term))$

route2():=(
     wbp:0,sumkj:0,sumk:0,
     for idx:itl thru itr do(phi(b,idx),
     dfp:(pml**2-p0[idx]**2),
     termk:2*pml/dfp,
     sumk:sumk+termk,
     wbp:wbp+phi**2/dfp,
     sumj(idx,pml),
     sumkj:sumkj+phi**2*sumj/dfp),
     exp0:1-(n/ntr)*wc*wbp,
     f[0]:ev(exp0,p:pml),
     exp1:sumk-(n/ntr)*dwc*wbp-(n/ntr)*wc*sumkj,
     f[1]:ev(exp1,p:pml))$
```

APPENDIX II - Listing of MACSYMA routine FREE_BEAM.MAC

```
comment():=(print("this module contains the functions needed
          to specify the basis functions and characteristic
          parameters of a freely suspended uniform Euler-
          Bernoulli beam. The beam constants are -
          length, el, and EI/mA, which are defined at set-
          up time. The coordinate variable, x, and the mode
          index, argk, are specified as calling arguments of

          the functions phi(x,argk) and startp(argk)"))$

base_structure():=(comment(),
          el:read("enter beam length"),
          eima:read("enter value for EI/mA"),
          eima2:sqrt(eima),
          eima4:sqrt(eima2),

print("baseline structure is a freely suspended
          Euler-Bernoulli beam. length = ",el," EI/mA =
          ",eima))$

phi(x,argk):=(mu(argk),
     ml:mu*el,mx:mu*x,shx:sinh(mx),shl:sinh(ml),
     sx:sin(mx),sl:sin(ml),chx:cosh(mx),
     chl:cosh(ml),cx:cos(mx),cl:cos(ml),
     shmsl:shl-sl,chmcl:chl-cl,
     shpsl:shl+sl,chpcl:chl+cl,
     shpsx:shx+sx,chpcx:chx+cx,
     phi:(2/eima4)*(shmsl*chpcx-chmcl*shpsx)/
             (shmsl*chpcl-chmcl*shpsl),phi)$

startp(argk):=(mu(argk), val:eima2*mu**2,
     p0[argk]:%i*val)$

mu(argk):=(
     k2:%pi*(argk+1/2),cs:cosh(k2),
     mu:(k2*cs-(-1)**argk)/cs,mu)$
```

APPENDIX III - Listing of MACSYMA routine MLTMAS.MAC

```
comment():=(print("this module derives the transfer function of
   a local modification consisting of a series of lumped
   mass/spring/dampers which may or may not be attached to
   ground, depending on the values assigned to the components
   (see documentation). the output of this module are:
        wc =wc(p) - the impedance function of the substructure
        as seen at the point of attachment (driving point
        impedance),
        numwc - the numerator of the rational form of wc,
        denwc - the denominator of the rational form of wc, and
        dwc - the derivative of wc with respect to p "))$

   sub_structure():=(comment(),input(),process(),printout())$

   input():=(nd:read("enter order of substructure"),
        print("enter substructure parameters as requested"),
        for id:1 thru nd do(
        m[id]:read(mass[id]),k[id]:read(spring[id]),
        c[id]:read(damper[id])))$

   process():=block(ratprint:false,
        nd1:nd-1,if nd1<1 then (xx[2]:0,go(cont)),
        xx[nd]:(k[nd1]+c[nd1]*p)/(m[nd]*p**2+
        (c[nd1]+c[nd])*p+k[nd1]+k[nd]),
        if nd1<2 then go(cont),
        for id:nd1 step -1 thru 2 do(
        id1:id-1,
        xx[id]:(k[id1]+c[id1]*p)/(m[id]*p**2+
        (c[id1]+c[id]*(1-xx[id+1]))*p+
        k[id1]+k[id]*(1-xx[id+1]))),cont,
        wc:ratsimp(-(k[1]+c[1]*p)*(1-xx[2])-m[1]*p**2,p),
        dwc:diff(wc,p),
        numwc:num(wc),
        denwc:denom(wc))$

   printout():=(print("transfer function wc(p) = ",wc),
        print("numerator of wc(p) =",numwc),
        print("denominator of wc(p) =",denwc),
        print("set up of substructure transfer function
        complete"))$
```

APPENDIX IV - Copy of paper from the Proceedings of the International Conference on Noise & Vibration '89, Singapore, August 16-18, 1989.

International Conference
NOISE & VIBRATION '89

ANALYSIS OF MODES AND FREQUENCIES OF MODIFIED STRUCTURES USING COMPUTER ALGEBRA

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INTRODUCTION

During the development stages of structural systems such as aircraft, rotorcraft or spacecraft, it is usually of interest to analyze the effects of modifications on the dynamic response of the system. Sometimes these modifications are being proposed as solutions to some vibration problem; at other times, the exercise is part of a parametric study aimed at optimizing the vibration characteristics of the structure. As can be seen from a recent review of this subject area [1], most of the current approaches to this problem are based on numerical techniques such as the finite element method, which permits the calculation of the new characteristic frequencies and modeshapes resulting from the changes. Direct analytical approaches have been attempted only for very simple structural configurations such as beams [2] and plates [3], or for local modifications [4],[5]. Other methods that have been reported in the literature involve the direct calculation of the changes to frequency response functions (e.g. mobilities) which have been measured from the baseline structure [6].

The major obstacle to analytical approaches to structural modifications is the complexity of the algebraic manipulations required. A recent paper [7], is an illustration of how intractable the equations can become, even in cases when simple beams and plates are being considered. Using a formal approach developed in Ref. [8], this paper presents a method for automating the derivation of the Green's functions of modified structures using computer algebra (MACSYMA). MACSYMA has a wide range of algebraic-manipulation capabilities, all working on symbolic inputs and yielding symbolic results [9],[10]. Starting with the Green's function of the baseline structure, this method utilizes computer algebra to derive the Green's function of the modified structure when these modifications are in the form of local attachments of lumped-parameter subsystems or

controllers. The primary advantage of this approach over the usual numerical techniques, is the availability of analytical functions for performing such design tasks as optimization and parametric studies.

MODELLING OF INTERCONNECTED STRUCTURES

Butkovskyi's Method. Let the response of a general distributed parameter elastic structure to excitation $w(x_1,t)$ be $Q(x_2,t)$; $x_1 \in M_1$, $x_2 \in M_2$, $t \in \Omega$, where M_1 and M_2 are the sets of the space variables of the excitations and responses respectively. Ω is an interval of time $[t_0,t_1]$;

$$L_{x,t}(Q(x_2,t)) = w(x_1,t) \tag{1}$$

where $L_{x,t}()$ is a partial differential operator in space coordinates x and time coordinate t, subject to the appropriate boundary and initial conditions. Following Ref. [8], an elementary distributed block can be defined for this structure as shown in Fig. 1.

$$w(x_1,t) \longrightarrow \boxed{G(x_2,x_1,t,\tau)} \longrightarrow Q(x_2,t)$$

Figure 1. Linear Distributed Block

The Green's function $G(x_2,x_1,t,\tau)$ satisfies the equation:

$$L_{x,t}(G(x_2,\xi,t,\tau)) = \delta(x_1-\xi)\delta(t-\tau) \tag{2}$$

subject to the same boundary and initial conditions as Eq.(1). $\delta()$ is the Dirac-delta function [11]. It follows that for stationary distributed blocks;

$$Q(x_2,t) = \int_0^t \int_{D_1} (G(x_2,\xi,t-\tau)w(\xi,\tau))\,d\xi d\tau \tag{3}$$

The Green's function, as defined in Eq.(2) and Eq.(3), is also commonly referred to as the impulse response function of the structure. Its Laplace transform is the transfer function $W(x_2,\xi,p)$, p being the complex valued Laplace variable:

$$p = \alpha + i\omega \tag{4}$$

For stationary distributed blocks, the schematic of Fig. 1, is often used with the transfer function in place of the Green's function, and ofcourse the input and output quantities are replaced by their respective Laplace transforms. Given the transfer function, the Green's function is obtainable by performing the inverse Laplace transform.

The formal rules for deriving the Green's functions of interconnected distributed blocks are as follows:

(a) Blocks connected in parallel:

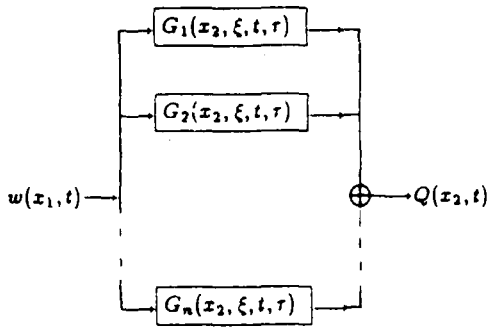

Figure 2. Parallel Connection

The Green's function for the combined system of Fig. 2 is:

$$G(x_2,\xi,t,\tau)=\sum_{i=1}^{n}G_i(x_2,\xi,t,\tau) \tag{5}$$

(b) Blocks connected in series:

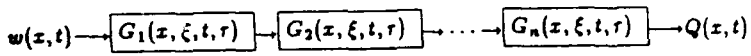

Figure 3. Series Connection

The Green's function for the combined system of Fig. 3 is:

$$G(x,\xi,t,\tau)=$$

$$\underbrace{\int_0^t\cdots\int_0^t}_{n-\text{times}}\left\{\int_{D_{n-1}}\cdots\int_{D_1}\left[\prod G\right]d\eta_1\cdots d\eta_{n-1}\right\}d\tau_1\cdots d\tau_{n-1} \tag{6}$$

with the definition,

$$\prod G\equiv G_n(x,\eta_1,t,\tau_1)G_{n-1}\quad(\eta_1,\eta_2,\tau_1,\tau_2)\cdots G_1(\eta_{n-1},\xi,\tau_{n-1},\tau)$$

(c) Feedback:

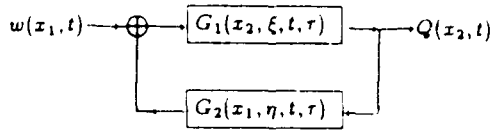

Figure 4. Feedback

The Green's function for the combined system of Fig. 4 is obtained as the solution to the following equation:

$$G(x_2,\xi,t,\tau)=G_1(x_2,\xi,t,\tau)+$$

$$\int_0^t\int_0^t\left\{\int_{D_2}\int_{D_1}\left|G_1(x_2,\xi_1,t,\tau_2)G_2(\xi_1,\eta,\tau_2,\tau_1)G(\eta,\xi,\tau_1,\tau)\right|d\xi_1 d\eta\right\}d\tau_1 d\tau_2 \tag{7}$$

Eq. (7) is an integral equation for the unknown function $G(x_2,\xi,t,\tau)$. For systems with finite dimensions, this equation is a Fredholm equation of the second kind. In order to represent the interaction between interconnected subsystems, special (intermediate) blocks are employed. Such blocks are used to achieve the transformation of signals from the domain of one subsystem to that of another at the regions (or points) of connection.

General Form for Green's Functions. Let L be a linear differential operator in a spatial domain M specified by the physical boundaries of the distributed parameter system. The response of the system Q(x), x M; can be expanded in terms of the orthonormal basis functions $\psi_k(x)$ such that:

$$Q(x)=\sum_{k=1}^{\infty}\alpha_k\psi_k(x) \tag{8}$$

where the basis functions are defined by the characteristic equation:

$$L\psi_k(x)=\lambda_k\psi_k(x) \tag{9}$$

and α_k is obtained as the dot product between Q(x) and $\psi_k(x)$, i.e.

$$\alpha_k=\int_M Q(x)\psi_k(x)dx \tag{10}$$

If the above definition of the basis functions is feasible, it follows that:

$$LQ(x) = \sum_{k=1}^{\infty} \alpha_k \lambda_k \psi_k(x) \tag{11}$$

and that:

$$L(LQ(x)) = L^2 Q(x) = \sum_{k=1}^{\infty} \alpha_k \lambda_k^2 \psi_k(x) \tag{12}$$

and also that:

$$L^n Q(x) = \sum_{k=1}^{\infty} \alpha_k \lambda_k^n \psi_k(x) \tag{13}$$

Hence, for an analytic function f(L);

$$f(L)Q(x) = \sum_{k=1}^{\infty} \alpha_k f(\lambda_k) \psi_k(x) \tag{14}$$

In order to invert the operator $L-\lambda$ for some complex number λ, f(L) can be identified as $1/(L-\lambda)$, so that:

$$(L-\lambda)^{-1} w(x) = \sum_{k=1}^{\infty} \alpha_k \frac{1}{\lambda_k - \lambda} \psi_k(x) \tag{15}$$

It is already known that if L is a differential operator, the inverse operator $(L-\lambda)^{-1}$ is represented by the transfer function $W(x,\xi,\lambda)$ as [12]:

$$(L-\lambda)^{-1} w(x) = \int_M W(x,\xi,\lambda) w(\xi) d\xi \tag{16}$$

where λ is treated as a parameter and the integration is taken over the appropriate spatial domain. This follows from the definition of the transfer function:

$$(L-\lambda) W(x,\xi,\lambda) = \delta(x-\xi) \tag{17}$$

So that,

$$\int_M W(x,\xi,\lambda) w(\xi) d\xi = (L-\lambda)^{-1} \int_M w(\xi) \delta(x-\xi) d\xi$$
$$= (L-\lambda)^{-1} w(x) \tag{18}$$

as stated.

This implies that:

$$\int_M W(x,\xi,\lambda) w(\xi) d\xi = \sum_{k=1}^{\infty} \alpha_k \frac{1}{\lambda_k - \lambda} \psi_k(x) \tag{19}$$

Considering a specific case of the preceding equation where $w(\xi_1)$ is the delta function $\delta(\xi_1 - \xi)$. Then the transfer function,

$$W(x,\xi,\lambda) = \int_M W(x,\xi_1,\lambda) \delta(\xi_1 - \xi) d\xi_1$$
$$= \sum_{k=1}^{\infty} \frac{\psi_k(x)\psi_k(\xi)}{\lambda_k - \lambda} \tag{20}$$

In problems pertaining to the dynamics of structures, the complex parameter is identified with the square of the laplace variable p, i.e.

$$\lambda = -p^2 \tag{21}$$

The Green's function for this class of problems is easily obtained as the inverse Laplace transform of the transfer function:

$$G(x,\xi,t) = \sum_{k=1}^{\infty} \frac{1}{ip_k} \psi_k(x)\psi_k(\xi) \sin ip_k t \tag{22}$$

The strategy for developing the routines that will automatically generate the Green's function of interconnected structural systems is as follows: (1) Derive the new transfer function using Butkovskyi's method; (2) Transform the new transfer function into the form of Eq. 20 (with consideration for Eq. 21) by deriving the new characteristic parameters and the new basis functions; (3) Repeat this process for each additional substructure; and (4) Obtain the Green's function of the complete system by using Eq. 22.

DISCRETE STRUCTURAL MODIFICATIONS

Block Diagram. When a lumped parameter substructure is attached at x=b, the structural diagram for the resulting system is shown in Fig. 5a, where the transfer function of the substructure (or controller), has been represented by $W_c(p)$. In this presentation, the effects of an active controller which applies some excitation to the structure in some relationship to a measured response is treated in much the same way as if a structural modification has been implemented locally. The transfer function of the controller replaces the impedance of the structural modification.

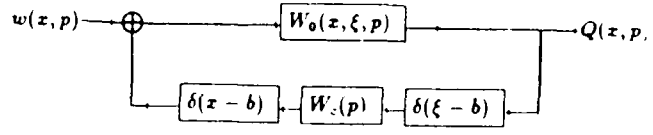

Figure 5a. Block diagram for local modification

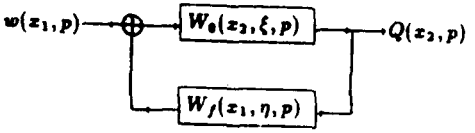

Figure 5b. Simplified block diagram

Note that in the feedback loop, the input signal into the modification block is the response of the structure at x=b, and the output signal out of the modification block is the excitation applied at x=b. The intermediate blocks containing Dirac-delta functions have been used to define the local values of the respective excitation and response functions. The structural diagram in Fig. 5a is a special case of the general feedback (see Fig. 5b). In the general case the new transfer function is defined by an integral equation:

$$W(x_2,\xi,p) = W_0(x_2,\xi,p) + \int_{M_2}\int_{M_1} W_0(x_2,\xi_1,p)W_f(\xi_1,\eta,p)W(\eta,\xi,p)d\xi_1 d\eta \qquad (23)$$

where $W_f(x, \xi, p)$ is obtained as a series connection of the blocks in the feedback loop. In case of the local attachment, the integral equation of Eq.23 reduces to an algebraic equation which can be rearranged to yield the new transfer function of the modified structure as:

$$W(x,\xi,p) = W_0(x,\xi,p) + \frac{W_0(x,b,p)W_0(b,\xi,p)}{1/W_c(p) - W_0(b,b,p)} \qquad (24)$$

Algebraic Algorithm for Modified Characteristic Parameters. For the modification of a distributed parameter baseline system by a discrete substructure, the new transfer function is given by Eq. 24. Let the transfer function of the baseline system be written in the form:

$$W_0(x,\xi,p) = \sum_{k=1}^{\infty} \frac{\psi_{0k}(x)\psi_{0k}(\xi)}{p^2 - p_{0k}^2} \qquad (25)$$

and that of the updated system be:

$$W_1(x,\xi,p) = \sum_{k=1}^{\infty} \frac{\psi_{1k}(x)\psi_{1k}(\xi)}{p^2 - p_{1k}^2} \qquad (26)$$

The characteristic parameters of the updated system p_{1k}, k=1,2,.. are the roots of the equation (by inspection of Eq. 24):

$$\frac{1}{W_c(p)} - W_0(b,b,p) = 0 \qquad (27)$$

or,

$$F(p) = \frac{1}{\sum_{k=1}^{\infty} \frac{(\psi_{0k}(b))^2}{p^2 - p_{0k}^2}} - W_c(p) = 0 \qquad (28)$$

Let

$$F_n(p) = \frac{1}{\sum_{k=1}^{\infty} \frac{(\psi_{0k}(b))^2}{p^2 - p_{0k}^2}} - n\varepsilon W_c(p) \qquad (29)$$

where $n = 1,2,....N; \varepsilon = 1/N$ and N is some large number which can be set as a program input at execution time. Let the roots of $F_n(p) = 0$ be denoted by $p_{1k}(n)$. Then $p_{1k}(n-1)$ is an approximation to $p_{1k}(n)$ if N is large enough, and $p_{1k}(n)$ can be obtained by the Newton-Raphson approximation as:

$$p_{1k}(n) = p_{1k}(n-1) - \frac{F_n(p_{1k}(n-1))}{F_n'(p_{1k}(n-1))} \qquad (30)$$

where $F'_n(p)$ is the derivative of $F_n(p)$ with respect to p. The algorithm is initiated by setting $p_{1k}(0) = p_{0k}$, a fact which follows directly from Eq. 29. For each of the k's, when n reaches N, the updated characteristic parameters are obtained:

$$p_{1k} = p_{1k}(N) \qquad (31)$$

Algebraic Algorithm for Modified Characteristic Functions. The advantages of seeking the updated transfer function in the form of Eq. 20 are two-fold. One is that it makes it easy to perform the inversion to obtain the updated Green's function, the second is that it allows for a recursive algorithm for considering as many updates of the system as is necessary to model the complete configuration. The preceding section showed how to derive the updated characteristic parameters. In this section, a formulation for deriving the updated basis functions is presented.

Let the k-th updated basis function be expanded in terms of the previous set of basis functions:

$$\psi_{1k}(x) = \sum_{n=1}^{\infty} \alpha_{kn}\psi_{0n}(x) \qquad (32)$$

The objective of this algorithm is to derive the coefficients α_{kn}, using information obtainable from the

characteristic parameters and the previous set of basis functions. Direct substitution of Eq. 32 into Eq. 26 gives:

$$W_1(x,\xi,p) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\alpha_{kn}\alpha_{km}\psi_{0n}(x)\psi_{0m}(\xi)}{p^2 - p_{1k}^2} \tag{33}$$

Substituting the new expression for $W_1(x, ,p)$ given by the right hand side of Eq. 24 (RHS_{24}) into Eq. 33, and utilizing the orthonormality of the basis functions in the right hand side (RHS_{33}) of Eq. 33, the following integral operations can be performed on both sides of Eq. 33 to get:

$$\int_M\int_M RHS_{24}\,\psi_{0n}(x)\psi_{0n}(\xi)dxd\xi = Z_n(p) \tag{34}$$

and

$$\int_M\int_M RHS_{33}\,\psi_{0n}(x)\psi_{0n}(\xi)dxd\xi = \sum_{k=1}^{\infty}\frac{\alpha_{kn}^2}{p^2 - p_{1k}^2} \tag{35}$$

By virtue of the equivalence of RHS_{33} and RHS_{24}, it follows that:

$$\sum_{k=1}^{\infty}\frac{\alpha_{kn}^2}{p^2 - p_{1k}^2} = Z_n(p) \tag{36}$$

One way of obtaining α_{kn} from Eq. 36 is to express $Z_n(p)$ as a ratio of polynomial functions of p,

$$Z_n(p) = \frac{{}_N Z_n(p)}{{}_D Z_n(p)} \tag{37}$$

Let,

$${}_D Z_{kn}(p) = {}_D Z_n(p)/(p^2 - p_{1k}^2) \tag{38}$$

Then,

$$\alpha_{kn}^2 = \lim_{p\to p_{1k}} \frac{{}_N Z_n(p)}{{}_D Z_{kn}(p)} \tag{39}$$

APPLICATION OF COMPUTER ALGEBRA

The computer algebra (e.g. MACSYMA) programming environment allows for the definition, storage, manipulation, simplification and evaluation of symbolic variables, functions and operations as well as numerical variables. Using computer algebra, all the algebraic operations presented in the preceding two sections can be coded such that the starting basis functions are input as functions of the spatial variables and the corresponding characteristic parameters are input values. The sequential updates of the system by attachment of each additional substructure can also be defined in terms of the functions representing the impedance function of the update, and the

location of attachment (also in symbolic form if necessary). With computer algebra, the results of all the mathematical operations that are performed on the functions and variables are retained in the simplest possible symbolic form. Any number of new functions can be defined to represent the result of multiple recursive operations. If and when desired, numbers can be substituted for all the unknown input variables, and the final result can be obtained in numerical form.

CONCLUSIONS

This paper has presented the formulation of an approach to the analysis of interconnected structures using computer algebra. Based on the general form of the Green's functions, computer algebraic algorithms have been formulated which permit the derivation of the basis functions (modes) and characteristic parameters of modified structures resulting from the attachment of discrete substructures to a base line structure. By treating complex interconnected structures as the result of incremental modifications of a baseline structure, the method presented here can be used to derive the Green's functions for such systems.

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REFERENCES

[1] Baldwin, J.F., Hutton, S.G., "Natural Modes of Modified Structures", AIAA Journal, Vol. 23, No. 11, Nov. 1985, pp. 1737-1743.

[2] Young, D., "Vibration of a Beam with Concentrated Mass, Spring, and Dashpot," Journal of Applied Mechanics, Vol. 15, 1948, pp. 65-72.

[3] Das, Y.C. and Navaratna, D.R., "Vibrations of a Rectangular Plate with Concentrated Mass, Spring and Dashpot," Journal of Applied Mechanics, Vol. 30, March 1963, pp. 31-36.

[4] Weissenburger, J.T., "Effect of Local Modifications on the Vibration Characteristics of Linear Systems," Journal of Applied Mechanics, Vol. 35, June 1968, pp. 327-332.

[5] Pomazal, R.J. and Snyder, V.W., "Local Modifications of Damped Linear Systems," AIAA Journal, Vol. 9, Nov. 1971, pp.2216-2221.

[6] Flannelly, W.G., Nagy, E.J., Fabunmi, J.A., "Analytical Testing", NASA CR-4329, May 1981.

[7] Kelkel, K., "Green's Functions and Receptance for Structures consisting of Beams and Plates," Paper No. 86-0927, AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, San Antonio, Texas, May 1986, Part 2, pp.376-385.

[8] Butkovskiy, A.G., Structural Theory of Distributed Systems. Ellis Horwood Ltd., 1983.

[9] Rand, R.H., Computer Algebra in Applied Mathematics: An
Introduction to MACSYMA, Pitman, 1984.

[10] Pavelle, R., Editor, Applications of Computer Algebra, Kluwer, 1985.

[11] Roach, G.F., Green's Functions: Introductory Theory with Applications. Van Nostrand, 1970.

[12] Keener, K.P., Principles of Applied Mathematics: Transformation and Approximation. Addison-Wesley, 1988.